Congruence: same shape, same size

Core learning - Foundation and Higher

What does congruent mean?

Congruent shapes fit exactly on top of one another. Corresponding sides have equal lengths and corresponding angles are equal.

A shape stays congruent after a translation, rotation or reflection.

An enlargement with scale factor 2 changes its size, so it is not congruent.



These triangles are congruent. The second has been reflected.

The vertex order matters: triangle ABC corresponds to triangle DEF.

AB matches DE; AC matches DF; BC matches EF.

Four tests for congruent triangles

SSS: all three corresponding sides are equal.

SAS: two sides and the included angle are equal.

ASA / AAS: two angles and a corresponding side are equal.

RHS: both are right-angled, with equal hypotenuses and one other side.

Important words

Included angle: the angle between the two specified sides.

Hypotenuse: the side opposite the right angle; it is the longest side.

Avoid this mistake

AAA proves the same shape, but does not prove the same size.

Two sides and a non-included angle (SSA) are not a general congruence test.

Using the congruence tests

Core methods, followed by a Higher-tier proof

Worked example 1 - choosing the correct test

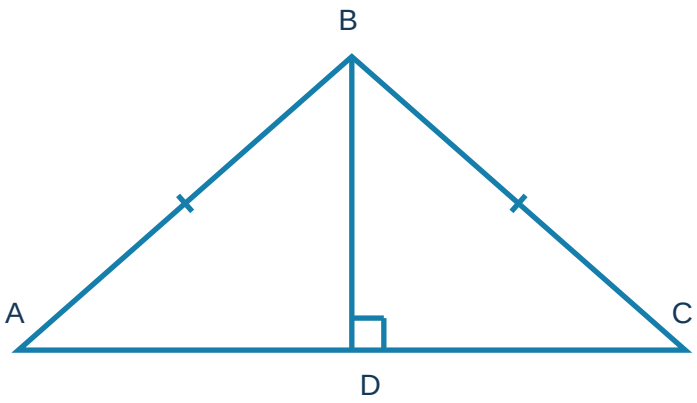
One triangle has sides 6 cm and 9 cm with a 40-degree angle between them.
A second triangle has the same two sides and the same included angle.
The triangles are congruent by SAS. All their matching sides and angles agree.

Worked example 2 - two angles and a side

Triangle PQR has angles P = 48 degrees and Q = 67 degrees, with PQ = 8 cm.
Triangle XYZ has X = 48 degrees and Y = 67 degrees, with XY = 8 cm.
They are congruent by ASA. Their third angles are both:
180 - 48 - 67 = 65 degrees. Therefore angle R = angle Z = 65 degrees.

Higher extension - writing a proof

In the diagram, AB = BC and BD is perpendicular to AC.
Prove that triangles ABD and CBD are congruent.



1. Angle ADB = angle CDB = 90 degrees, because BD is perpendicular to AC.
2. AB = CB, as given. These are the hypotenuses of the right triangles.
3. BD = BD, because the triangles share this side.
4. Therefore triangles ABD and CBD are congruent by RHS.
5. Matching parts are equal, so AD = DC and angle ABD = angle CBD.

Proof checklist

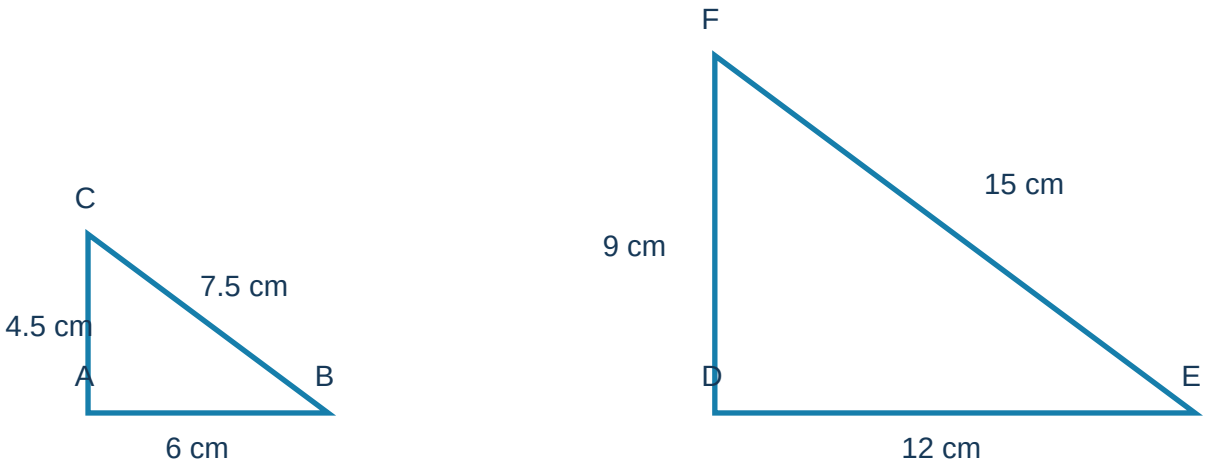
State each equality and its reason, then name the congruence test.
Use given information and angle facts; do not rely on how a drawing looks.

Similar shapes and matching sides

Core learning - Foundation and Higher

What does similar mean?

Similar shapes have the same shape. Their sizes may be different. Corresponding angles are equal. Every corresponding length is multiplied by the same positive scale factor. Similar shapes may be turned or reflected. Congruent shapes are a special case of similar shapes with scale factor 1.



Match A to D, B to E and C to F. All three side ratios are equal:
DE / AB = 12 / 6 = 2; DF / AC = 9 / 4.5 = 2; EF / BC = 15 / 7.5 = 2.
The scale factor from ABC to DEF is 2. From DEF to ABC it is 1/2.

How to recognise similar triangles

AA: two corresponding angles are equal; the third angles must also match.
SSS similarity: all three corresponding side ratios are equal.
SAS similarity: two corresponding side ratios are equal AND the included angles are equal. These are similarity tests, not congruence tests.

Scale factor and missing lengths

Scale factor k = length in the new shape / matching length in the original.
New length = original length x k.
Original length = new length / k.

Match carefully

Match vertices using equal angles; then match the sides between them.
Do not pair sides just because they occupy the same place on the page.

Similarity: worked length examples

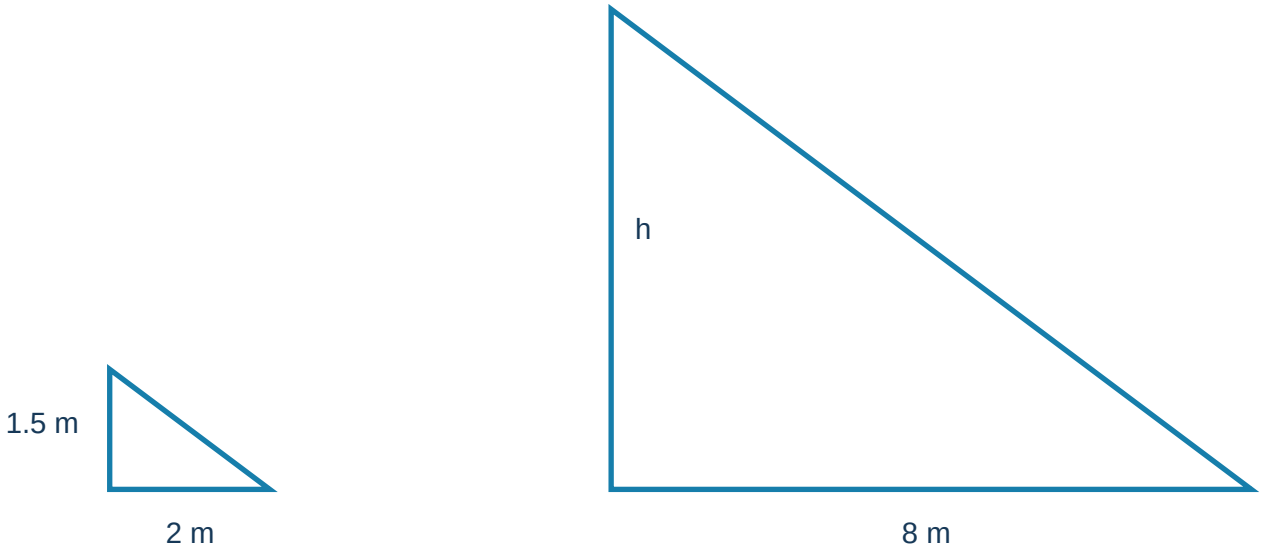
Core methods - Foundation and Higher

Example 1 - enlarging and reducing

Two similar rectangles have widths 5 cm and 15 cm.
Scale factor from the smaller to the larger = 15 / 5 = 3.
The smaller rectangle is 8 cm long, so the larger length = 8 x 3 = 24 cm.
If a matching length on the larger shape is 21 cm, the smaller is 21 / 3 = 7 cm.

Example 2 - finding a height from shadows

A vertical 1.5 m pole casts a 2 m shadow. At the same time, a vertical tree on the same level ground casts an 8 m shadow. Find the height of the tree.



Both triangles have a right angle. The sun rays make equal angles with the ground, so the triangles are similar by AA.
Scale factor = 8 / 2 = 4. Tree height = 1.5 x 4 = 6 m.

Example 3 - perimeter

A shape has perimeter 18 cm. A similar shape has length scale factor 2.5.
Every side is multiplied by 2.5, so the perimeter is also multiplied by 2.5.
New perimeter = 18 x 2.5 = 45 cm.

Keep units consistent

Convert matching lengths to the same unit before calculating a ratio.
For example, 1.2 m / 40 cm = 120 cm / 40 cm = 3.

Higher: areas, volumes and proofs

Higher-tier extension - learn after the core methods

Length, area and volume scale factors

For similar shapes or solids with length scale factor k:
Lengths and perimeters scale by k. Areas scale by k squared.
Volumes scale by k cubed. Use corresponding measurements in the same units.

Example 1 - area

Similar triangles have corresponding sides 4 cm and 10 cm.
Length scale factor = 10 / 4 = 2.5.
If the smaller area is 12 square cm, the larger area is:
12 x 2.5 squared = 12 x 6.25 = 75 square cm.

Example 2 - working backwards from area

Two similar shapes have areas 20 square cm and 180 square cm.
Area ratio = 180 / 20 = 9. Length scale factor = square root of 9 = 3.
A side of 7 cm on the smaller shape corresponds to 7 x 3 = 21 cm.

Example 3 - volume

Similar solids have volume ratio small : large = 27 : 125.
Their length ratio is cube root of 27 : cube root of 125 = 3 : 5.
A 6 cm length on the smaller solid corresponds to 6 x 5 / 3 = 10 cm.

Proof method: a triangle inside a triangle

If D lies on AB, E lies on AC and DE is parallel to BC, then:
Angle ADE = angle ABC and angle AED = angle ACB (corresponding angles).
Therefore triangles ADE and ABC are similar by AA.
Hence AD / AB = AE / AC = DE / BC. Use whole sides AB and AC.

Final reminders

Congruent: equal corresponding lengths. Similar: equal length ratios.
For areas use k squared; for volumes use k cubed.
Explain why triangles are congruent or similar before using matching parts.